Structural Optimization with Flutter Speed Constraints Using Maximized Step Size

R.F. O'Connell,* N.A. Radovcich,† and H.J. Hassig‡
Lockheed-California Company, Burbank, Calif.

A procedure is presented for the minimization of structural mass while satisfying flutter speed constraints. The procedure differs from other optimization methods in that the flutter speed is exactly satisfied at each resizing step, and the step size is determined by a direct minimization of the objective function (mass) for each set of flutter derivatives calculated. In conjunction with this method, a new move vector is suggested which results in a very efficient resizing procedure.

I. Introduction

MANY procedures are available for the minimization of structural mass while satisfying flutter speed constraints, and certain characteristics are common to most methods. This is not surprising, since optimization for flutter is relatively simple in concept: structural mass (in the form of stiffness or mass balance) is added where it is most effective in raising the flutter speed and/or removed where it is least effective or lowers the flutter speed. In practice, however, the process is not so simple as the previous statement would indicate. It is well-known that the relationship between structural stiffness and flutter speed is highly nonlinear. Were this not the case, a formalized optimization procedure for flutter would hardly be necessary since only the most effective structural element would be increased and all other structural elements would be decreased until other constraints were encountered. In actuality, however, adding stiffness to the most effective element reduces the effectiveness of that element, which results in another element becoming most effective. If this new element is then stiffened, the process is repeated. If this procedure were to be repeated a sufficient number of times, using sufficiently small stiffness increments, a structure approximately optimized for flutter would result. For a design effort of practical interest, involving many stiffness elements or design variables in complex structural configurations, such an approach would be prohibitive in terms of time, cost and computational resources required. In such cases, more sophisticated procedures are necessary.

The available direct methods of flutter optimization, i.e., those methods which directly minimize the objective function, may be characterized in a number of ways, but for the purposes of the present paper it is convenient to classify them as either arbitrary step size or defined step size procedures. Arbitrary step size procedures are those in which the magnitude of the step at any point in the procedure is left to the discretion of the analyst. The flutter optimization usually involves a large number of small steps, requiring one flutter solution and one set of flutter speed derivatives per step. The methods of Simodynes[1] and Rudisill and Bhatia[2] are examples of this type of procedure. In contrast to these are the procedures in which step size criteria are explicitly defined. In these procedures, fewer steps are usually required than for the arbitrary step size procedures to achieve the same degree of optimization. However, several solutions of the characteristic flutter equation as well as one set of flutter derivatives are required for each step. Examples of this latter category are the penalty function procedures[3,4] and the feasible directions methods.[5]

The method presented here is a defined step size procedure. The method will be described and then applied to a simple flutter optimization task. First, however, one of the basic tools used in the procedure, utilizing a generalized form of the flutter equation, will be described.

II. The Generalized Flutter Equation

The flutter equation is usually expressed in matrix form, relating forces to deflections through the coefficients of mass, stiffness and aerodynamics parameters. In the *p-k* method,[6] for example, the flutter equation for a base configuration, defined by a mass matrix M_0 and a stiffness matrix K_0, is given by Eq. (1), where p is the complex characteristic

$$\left[\frac{V^2}{c^2}[M_0]p^2+(1+ig)[K_0]-\frac{1}{2}\rho V^2[A(ik)]\right]\{q\}=0 \quad (1)$$

variable $p=(\gamma+i)k$ and $A(ik)$ is the aerodynamics matrix which is a function of the reduced frequency $k=(\omega c/V)$; c is a reference length and ω is the circular frequency. In this application, the velocity (V), density (ρ) and structural damping (g) are specified and the real and imaginary parts of p are determined such that the characteristic determinant of Eq. (1) is equal to zero. This may be expressed as in Eq. (2), where D is the flutter determinant.

$$D((\gamma+i)k,g,V,\rho)=0 \quad (2)$$

Suppose now that it is desired to determine the increment in a design variable m_j such that a specified flutter speed is exactly satisfied. Assuming that the mass and stiffness matrices are linear functions of the design variable this may be expressed as in Eq. (3), where $[M_j]$

$$\left[\frac{V^2}{c^2}[M_0]p^2+(1+ig)[K_0]-\frac{1}{2}\rho V^2[A(ik)]\right.$$
$$\left.+m_j\left(\frac{V^2}{c^2}[M_j]p^2+(1+ig)[K_j]\right)\right]\{q\}=0 \quad (3)$$

and $[K_j]$ represents the mass and stiffness distributions of a unit increment of the design variable and m_j is the magnitude of the increment. In this case, the velocity (V), density (ρ), structural damping (g), and real part of p, e.g. $\gamma=0$, are specified and values of k and m_j are determined which satisfy

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*Department Manager, Aeromechanics Department. Associate Fellow AIAA.

†Research Specialist Senior. Member AIAA.

‡Research and Development Engineer. Member AIAA.

the equation. This is expressed in more general form in Eq. (4), which represents an extension of Incremented Flutter Analysis.[7]

$$D((\gamma+i)k,g,V,\rho,m_j)=0 \tag{4}$$

Equation (4) can be solved directly for m_j and k by means of the Two-Dimensional Regula Falsi method,[8] given initial trial values for m_j and k.

III. Flutter Optimization Method

For the purposes of illustrating the proposed method, a structure will be assumed which is representative of a structure designed by strength considerations only and which is initially deficient in flutter speed margin. It is desired to determine the combination of design variables m_j measured from strength design levels, which will satisfy the flutter speed requirement with the minimum of added mass. As a first step, the flutter speed will be increased to the required value.

Initial Flutter Speed Increase

The flutter speed may be increased by specifying any desired distribution of design variables and then determining the amount of the design variable distribution required to increase the flutter speed to exactly the desired value by means of the generalized flutter equation.

Although any desired distribution of design variables may be used to increase the flutter speed, it is clear that the use of a distribution resulting in a nearly optimum structure would tend to minimize the required subsequent resizing. For this purpose, a distribution of design variables based on the gradient of flutter velocity with respect to the design variables will accomplish the increase in flutter speed. Each design variable is proportional to the corresponding flutter velocity derivative[§] [Eq. (5)], where the flutter velocity derivatives are formed

$$\{m_j\}=a\{\partial V/\partial m_j\} \tag{5}$$

in the manner of Ref. 2. It should be noted that the use of the velocity gradient to determine the resizing distribution (move direction) assumes that the design variables are expressed in terms of weight or mass units, and that the velocity gradient is expressed in terms of velocity increments per unit weight or mass. Since the objective is to minimize total mass, the design variables in the present paper are specifically identified in term of mass units rather than in terms of the more general units of Ref. 2.

Substituting Eq. (5) into Eq. (4) leads to Eq. (6) with the two unknowns a and k.

$$D((\gamma+i)k,g,V,\rho,a\{\partial V/\partial m_j\})=0 \tag{6}$$

If V is the required flutter speed the solution for a defines the magnitude of the design change in the "direction" of the velocity gradient needed to increase the flutter speed to the required value.

Mass Reduction at Constant Flutter Speed

Once the desired flutter speed is obtained, mass reductions at a constant flutter speed are performed in discrete structural resizing steps in which the design variables are incremented from the previous structural resizing. As indicated previously, the process is one of adding mass (stiffness) to those design variables more effective in increasing flutter speed and removing mass (stiffness) from those design variables less effective in increasing flutter speed. In order to accomplish this a distribution of positive design variable increments, which is efficient in increasing the flutter speed, must be combined with a suitable negative distribution while keeping the flutter speed constant. For the positive distribution, the velocity gradient, used in Eq. (5), is again chosen. For the negative distribution a number of flutter optimization methods employ a distribution defined by the mass gradient.[2] Accordingly, the combined column of design variable increments is as shown in Eq. (7)

$$\{\Delta m_j\}=a\left\{\left\{\frac{\partial V}{\partial m_j}\right\}-b\{1\}\right\} \tag{7}$$

where the value of the coefficient b, given by Eq. (9), is determined such that the linearized change in flutter speed [Eq. (8)] is equal to zero. The column of design variable increments

$$\Delta V=[\partial V/\partial m_j]\{\Delta m\}=0 \tag{8}$$

$$b=\frac{[\partial V/\partial m_j]\left\{\dfrac{\partial V}{\partial m_j}\right\}}{[\partial V/\partial m_j]\{1\}} \tag{9}$$

defined by Eq. (7) is equivalent to that of the gradient projection search at constant flutter speed proposed in Ref. 2.

It is recognized that the design variable increments described in the preceding will not result in a flutter speed which remains exactly constant. Usually the additive increments will produce an increase in flutter speed which is less than that predicted by linear theory, while the subtractive increments will decrease the flutter speed more than predicted, and the flutter speed will decrease. In conjunction with the design variable increments [Eq. (7)] based on the linearized flutter equation, an adjustment is required in order to maintain a constant flutter speed. For this purpose, the positive or additive elements of the linearly derived distribution are used [Eq. (10)] where α defines the magnitude. The resizing

$$\{\Delta m_j''\}=\alpha\left(\text{positive elements of }\left\{\left\{\frac{\partial V}{\partial m_j}\right\}-b\{1\}\right\}\right) \tag{10}$$

procedure may then be described in terms of these two columns of design variable increments as follows: A one-dimensional search is conducted in which for selected magnitudes a of the column of increments in Eq. (7), the magnitude of the adjustment column [Eq. (10)] which maintains the flutter speed exactly constant is determined by use of the generalized flutter equation. As the magnitude of the linearized column of increments increases, the mass associated with the design variables decreases linearly. The mass of the design variables associated with the adjustment column, however, increases at a greater than linear rate. For some magnitude of the linearly determined column of increments, the total mass associated with this column combined with the adjustment column will be a minimum, and at that point one resizing step is terminated.

In practice, an equivalent but computationally simpler formulation is used. A distribution of negative design variable increments is formed as indicated in Eq. (11), and the magnitude, defined by β, is used as an independent variable.

$$\{\Delta m_j''\}=\beta\left(\text{negative elements of }\left\{\left\{\frac{\partial V}{\partial m_j}\right\}-b\{1\}\right\}\right) \tag{11}$$

The magnitude α of the column of positive increments $\{\Delta m_j''\}$ is then determined, by use of the generalized flutter equation, for each value of the independent variable such that the flutter speed remains constant. As before, one resizing step consists of a one-dimensional search to determine the minimum value of the total mass of the design variables. This latter formulation has the advantage that the design variable in-

[§] To demonstrate the approach taken in this paper it is convenient to assume all $\partial V/\partial m_j>0$. Generalization to include configurations for which there are negative $\partial V/\partial m$'s is straightforward.

crements are separated into positive and negative components with no common elements, making it somewhat simpler to deal with minimum size constraints. These minimum size constraints are only affected by the choice of the independent variable β so that the distribution or direction corresponding to the negative increments can be easily modified as these constraints are encountered.

Improved Move Vector

In the preceding description of the minimization of mass at constant flutter speed, a distribution of design variable increments for the addition of mass is combined with a distribution for the removal of mass [Eq. (7)]. The additive distribution, defined by the flutter velocity gradient, is widely used in flutter optimization procedures, and is a reasonably efficient distribution for increasing flutter speed. The subtractive component, however, removes mass from the design variables uniformly, and only performs the function of compensating for the flutter speed increase resulting from the additive component. This subtractive component is relatively inefficient in redistributing the design variable increments to the minimum mass configuration, since it results in the indiscriminate removal of mass. It is clear that if it is desired to remove mass in such a way as to maintain as high a flutter speed as possible, the mass gradient distribution should not be used. Rather, a distribution which removes the most mass from the least efficient design variables should be devised. A simple distribution which meets this requirement is one based on the reciprocals of the flutter velocity derivatives. Replacing the mass gradient in Eq. (7) by the reciprocal of the flutter speed derivatives leads to Eq. (12).

$$\{\Delta m_j\} = a\left\{\{\partial V/\partial m_j\} - b\left\{\frac{1}{\partial V/\partial m_j}\right\}\right\} \tag{12}$$

The scalar quantity b is again determined such that the linearized flutter velocity change is equal to zero [Eq. (13)]. The quantity N in

$$b = \frac{\lfloor \partial V/\partial m_j \rfloor \left\{\frac{\partial V}{\partial m_j}\right\}}{\lfloor \partial V/\partial m_j \rfloor \left\{\frac{1}{\partial V/\partial m_j}\right\}} = \frac{\lfloor \partial V/\partial m_j \rfloor \left\{\frac{\partial V}{\partial m_j}\right\}}{N} \tag{13}$$

this latter equation is an integer representing the number of design variables used, and it can be seen that b is then equal to the average of the square of the flutter speed derivatives. It is clear that this distribution of design variable increments could replace the distribution shown in Eq. (7), and that resizing steps based on this distribution would lead to a minimum-mass distribution. Both components of the distribution are effective in redistributing the mass, with the result that fewer resizing steps are required to achieve a given level of mass reduction.

It should be noted that the distribution suggested here for the removal of mass – proportional to the reciprocals of the flutter velocity derivatives – is only one of many possible distributions which might accomplish this removal efficiently. Other distributions might be devised which are more efficient than the one proposed. The important consideration, however, is that the mass removal should be accomplished in a manner which considers the effect of the design variables on the flutter speed.

IV. Numerical Example

In order to illustrate the proposed method, a simple optimization task will be demonstrated. For this purpose, a subsonic transport airplane was defined, having a reduced wing torsional stiffness assumed to be representative of a design based on strength considerations only. The structural model used is an EI, GJ beam representation having eight finite elements on the wing semi-span, numbered from inboard to outboard. The flutter analysis of this configuration indicates a flutter speed in the primary wing bending-torsion mode of approximately 440 KEAS. It was desired to increase the flutter speed to 525 KEAS with a minimum increase in mass, using the eight torsional stiffness elements of the wing as design variables. The initial values of the torsional stiffness elements were taken as the minimum allowable values of those elements.

First, a set of design variables must be determined which increases the flutter speed to the required 525 KEAS. Although it is proposed to use a distribution of the design variables proportional to the velocity gradient, it was decided that a less efficient distribution should be used for the test case in order to provide a more significant resizing task at the required flutter speed. Accordingly, a distribution proportional to the initial torsional stiffness was chosen, and the magnitude of

Table 1 Values of the design variables for the one-dimensional search in the first resizing step; distribution of Eq. (12)

Step[a] size lb	Values of the design variables, lb								Total mass of the design variables, Σm_j
	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	
0	249.7	231.7	199.0	144.4	140.2	68.8	48.0	33.7	1115.4
−500	45.3	88.0	106.9	117.8	147.1	116.4	97.7	0.4	719.6
−700	0	3.5	52.7	102.2	151.8	148.5	130.8	0	589.5
−760	0	0	8.8	89.6	154.3	165.7	148.4	0	566.8
−780	0	0	0	78.4	155.4	173.7	156.6	0	564.1
−800	0	0	0	58.4	157.0	184.4	167.6	0	567.4
−850	0	0	0	8.4	162.3	220.3	204.4	0	595.4

[a] Total mass increment of negative component, $\Sigma \Delta m_j^n$.

Table 2 Values of the design variables for three resizing steps; distribution of Eq. (12)

Step	Values of the design variables, lb								Total mass of the design variables, Σm_j lb
	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	
0	249.7	231.7	199.0	144.4	140.2	68.8	48.0	33.7	1115.4
1	0	0	0	78.4	155.4	173.7	156.6	0	564.1
2	0	0	0	69.3	158.5	165.8	133.5	27.1	554.2
3	0	0	0	69.4	176.8	169.6	122.6	13.0	551.4

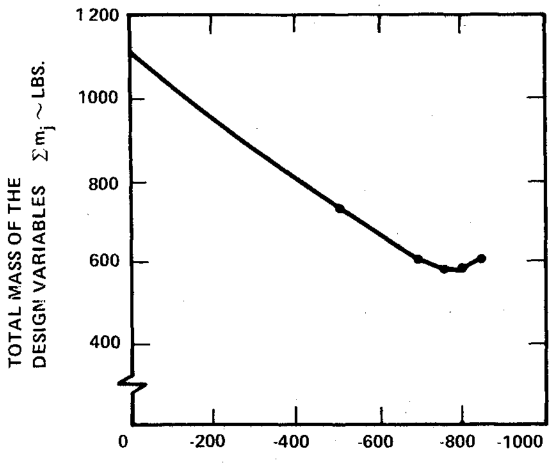

Fig. 1 Total mass of the design variables vs step size: distribution of Eq. (12).

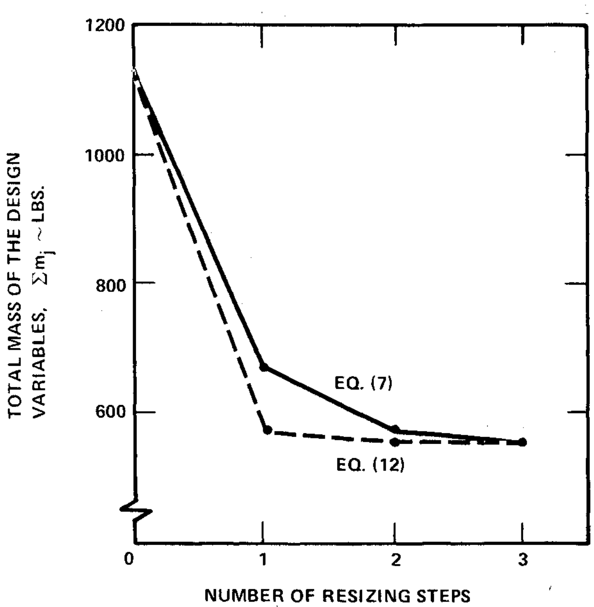

Fig. 2 Total mass of the design variables vs number of resizing steps; comparison between distributions of Eqs. (7) and (12).

that distribution was determined by the use of Eq. (3) which increased the flutter speed to 525 KEAS, requiring a weight increase of 1115 lb.

With this configuration as a starting point, a minimization of mass at constant flutter speed was performed using the distribution of design variable increments indicated in Eq. (12). For the first resizing step, results of the one-dimensional search to determine step size are shown in Fig. 1. The values of the design variables obtained during this search are given in Table 1. The step size is expressed in terms of the total mass associated with the negative component of the design variable increments Δm_j^n and the method of handling minimum size constraints can be seen from the distributions in Table 1. The results of the first three resizing steps are shown in Fig. 2. The values of the design variables for these three steps are given in Table 2. For comparison, the minimum mass achieved for this configuration, using an engineer-in-the-loop resizing procedure with a large number of steps, is 550.4 lb. Examination of these results shows that the method is very powerful, as demonstrated by the mass reduction achieved in the first resizing step, which is 98% of the difference between the initial mass and the minimum of 550.4 lb. The second and third step each reduce the mass by 73% of the remaining difference. In contrast to some other methods this monotonic mass reduction is characteristic of the proposed method.

In order to evaluate the effect of the choice of distribution of design variable increments on the efficiency of the resizing procedure, the resizing steps described in the preceding were repeated using Eq. (7). The results are shown in Fig. 2, along with the results obtained using Eq. (12). As can be seen, there is a substantial difference in mass reduction at the end of the first step, and the second step is still significant. For the optimization task demonstrated here, it is clear that either distribution would give satisfactory results. On the basis of the results, however, it is concluded that the distribution represented by Eq. (12) would produce a significantly improved performance in a practical flutter optimization task.

V. Discussion

The procedure presented is a direct application of the simple optimization concept: "add where it does the most good; subtract where it does the least harm." In that respect it resembles the method of Ref. 9. The latter method, however, is an indirect method of flutter optimization, i.e., the objective function is minimized by aiming at satisfying an optimality criterion. In methods other than the present one and the one of Ref. 9 the simple optimization concept is less easily recognizable.

The new procedure formally applies equally well to modalized and unmodalized flutter equations. For the numerical example the flutter equation is written in terms of 39 discrete displacement-type coordinates and is not modalized. Modalization aims at reducing computer cost by reducing the number of degrees of freedom. This reduction, however, must be compared with increased cost due to the modalization itself. Such increased costs are mainly associated with the repeated updating of vibration modes as the resizing progresses (assuming that modalization is by means of vibration modes). It should be noted that these considerations regarding modalization are equally valid for all published methods of optimization.

VI. Conclusions

The procedure presented here for the minimization of structural mass while satisfying flutter speed constraint appears to have a number of advantages over comparable methods: 1) after the initial step to increase flutter speed, the flutter speed constraint is satisfied exactly at each step, as well as at each substep in the one-dimensional minimization; 2) the one-dimensional minimization is performed directly on the objective function, mass, rather than on a related function; 3) the nonlinear relationship between flutter speed and the design variables is not only accounted for in the procedure but is used in achieving the maximum step size, or mass reduction, for each set of the flutter derivatives calculated; 4) the distribution of design variable increments is modified during the one-dimensional minimization to accommodate sizing constraints, thus making maximum use of each set of calculated flutter speed derivatives.

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